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Thermal Equilibration and Ellipsoidal Expansion of Rotationally-Symmetrical Longitudinal Flow in Relativistc Heavy Ion Collisions[1]

Feng Shengqin, Liu Feng and Liu Lianshou

Institute of Particle Physics, Huazhong Normal University, Wuhan, 430079, China

ABSTRACT

A rotationally-symmetrical ellipsoidal flow model is proposed for the relativistic heavy-ion collisions and compared with the 14.6 A GeV/c Si-Al and 10.8 A GeV/c Au-Au collision data. The large stopping in the heavier collision system and heavier produced particles is accounted for by using the ellipsoidal flow picture. The central dip in the proton and deuteron rapidity distributions for Si-Al collision are reproduced.

I. Introduction

The experimental finding that colliding nuclei are not transparent but undergo a violent reaction in central collisions represents one of the major motivations for the study of ultra-relativistic heavy ion collisions at the CERN/SPS, BNL/AGS and also at the future BNL/RICH and CERN/LHC. Of central importance is the ability of understanding to what extent the nuclear matter has been compressed and heated.

The study of collective flow in high energy nuclear collisions has attracted increasing attention from both experimental[1] and theoretical[2] point of view. The rich physics of longitudinal and transverse flow is due to their sensitivity to the system evolution at early time. The expansion and cooling of the heated and highly compressed matter could lead to considerable collectivity in the final state. Due to the high pressure, particles might be boosted in the transverse and longitudinal directions. The collective expansion of the system created during a heavy-ion collision implies space-momentum correlation in particle distributions at freeze-out.

The experimental data of the rapidity distributions of produced particles in 14.6 A GeV/c Si-Al collisions have been ultilized to study the collective expansion using a cylindrically-symmetrical flow model[3,4]. The model results fit well with the experimental distribution of pion, but is too narrow in the case of heavier particles proton and deuteron. In particular, the central dip, which can be clearly seen in the distribution of proton, is failed to be reproduced.

More recently, E877 Collaboration[5] has published their data for 10.8 A GeV/c Au-Au collisions, which provide a good chance to compare the stopping power in the collision systems of different sizes. The possible central peak of the rapidity distribution of proton at around mid-rapidity, which was obtained through extrapolating the experimental data to mid-rapidity using RQMD model[6], has been taken as an evidence for the increasing of stopping power, but the reliability of this extrapolation is model-dependent.

It has been shown earlier[7] that the ellipsoidal expansion is a simple way to take the nuclear stopping into account. In the present paper we propose a rotationally-symmetrical ellipsoidal flow model to describe the space-time evolusion in relativistic heavy ion collisions. The large stopping in the heavier collision system and heavier produced particles is described by using this picture. The central dips in the proton and deuteron rapidity distributions for Si-Al collisions are reproduced.

In section II the rotationally-symmetrical ellipsoidal flow model is formulated. The results of the model are given and compared with the experimental data in section III. A short summary and conclusions are given in section IV. In order to avoid the complexity in the production of strange particles and concentrate on the expansion of the system, we will discuss in this paper only normal non-strange particles —— pions, protons and deuterons.

II. Rotationally-symmetrical ellipsoidal flow

Firstly, let us briefly recall the fireball scenario of relativistic heavy ion collisions.

Since the temperature at freeze-out exceeds 100 MeV, the Boltzmann approximation is used. Transformed into rapidity y and transverse momentum p_t this implies[4]:

$$E\frac{\mathrm{d}^3n}{\mathrm{d}^3p} \propto E\mathrm{e}^{(-\mathrm{E/T})} = \mathrm{m_t}\cosh(\mathrm{y})\mathrm{e}^{(-\mathrm{m_t}\cosh(\mathrm{y})/\mathrm{T})} \tag{1}$$

Here $m_t = \sqrt{m^2+p_t^2}$ is the transverse mass, m is the mass of the produced particles at freeze-out.

The rapidity is defined as $y = \tanh^{-1}(p_l/E)$, where p_l is the longitudinal momentum of the produced particle. Substituting into Eq.(1) and integrating over m_t, we get the rapidity distribution of the isotropic thermal source,

$$\frac{\mathrm{d}n_{\mathrm{iso}}}{\mathrm{d}y} \propto \frac{m^2T}{(2\pi)^2}(1+2\xi_0+2\xi_0^2)\mathrm{e}^{(-1/\xi_0)}. \tag{2}$$

Here $\xi_0 = T/m\cosh(y)$.

However, the momentum distribution of the measured particles is certainly not isotropic. It is privileged in the direction of the incident nuclei. This is because the produced hadrons still carry their parent's kinematic information, making the longitudinal direction more populated than the transverse ones. The simplest way[3,4] to account for this anisotropy is to add the contribution from a set of fire-balls, sketched schematically in Fig.1 as dashed circles, with centers located uniformly in the rapidity region $[-y_0, y_0]$. The corresponding rapidity distribution is obtained through changing the ξ_0 in Eq,(2) into $\xi = T/m\cosh(y-y')$ and integrating over y' from $-y_0$ to y_0:

$$\frac{\mathrm{d}n_{\mathrm{cyl}}}{\mathrm{d}y} \propto \int_{-y_0}^{y_0}\mathrm{d}y'\frac{m^2T}{(2\pi)^2}(1+2\xi+2\xi^2)\mathrm{e}^{(-1/\xi)}, \tag{3}$$

$\xi = T/m\cosh(y-y')$. Equivalently, we can also use the angular variable Θ defined by $\Theta = 2\tan^{-1}\exp(-y')$, and change the integration variable in Eq.(3) to Θ, cf. the solid circle and lines in Fig.1.

This simple approach fits the rapidity distribution of pions well but failed to reproduce the central dip in heavier produced particles, which is clearly seen in the experimental distribution of protons and has some evidence in the distribution of deuterons.

Note that in this model the longitudinal and transverse expansions of the system are totally independent. This is a crude approximation. A more reasonable picture is an ellipsoidal expansion. For simplicity the rotational symmetry arround the longitudinal direction is still assumed, but the emission angle is now

In this model the ellipticity parameter e represents the degree of anisotropy of flow in the transverse and longitudinal direction. The smaller is e, the more anisotropic is the flow. The nuclear stopping can be taken into account in this way.

Subsituting Eq.(4) together with $y'_{\mathrm{e}} = -\ln\,\tan(\theta/2)$ into Eq.(3), the rapidity distribution is obtained:

$$\frac{\mathrm{d}n_{\mathrm{ellip}}}{\mathrm{d}y} = eKm^2T\int_{\theta_{\min}}^{\theta_{\max}}\left(1+\frac{2T}{m\cosh(y-y'_{\mathrm{e}})}+\frac{2T^2}{m^2\cosh^2(y-y'_{\mathrm{e}})}\right) \times\exp(-m\cosh(y-y'_{\mathrm{e}})/T)Q(\theta)\mathrm{d}\theta, \quad (5)$$

$$y'_{\mathrm{e}} = -\ln\,\tan(\theta/2)\ , \qquad Q(\theta) = \frac{1}{\sqrt{e^2+\tan^2\theta}|\cos\theta|\sin\theta}. \quad (6)$$

Here $\theta_{\min} = 2\tan^{-1}(e^{-y'_{\mathrm{e0}}})$, $\theta_{\max} = 2\tan^{-1}(e^{y'_{\mathrm{e0}}})$. y'_{e0} is the rapidity limit which confines the rapidity interval of ellipsoidal flow. We treat it together with the ellipticity e as two free papameters of the model to fit the amount of flow and stopping required by the data.

III. Comparison with experiments

The rapidity distributions of pion, proton and deuteron for 14.6 A GeV/c Si-Al collisions[9,10], are given in Fig.3 (a, b and c). The dashed, dotted and solid lines correspond to the results from isotropical thermal model, cylindrically-symmetrical flow model and rotationally-symmetrical ellipsoidal flow (RSEF) model respectively. The rapidity limit y'_{e0} and the ellipticity e used in the calculation are listed in Table I. The rapidity limit y'_0 used in the cylindrically-symmetrical flow model of Ref. [4] is also listed for comparison. The parameter T is chosen to be 0.12 GeV following Ref.[4].

Table I The value of model-parameters

Parameter	Si-Al Collisions			Au-Au Collisions	
	π	p	d	π	p
e	0.28	0.52	0.56	0.32	0.58
y'_{e0}	1.35	1.35	1.35	1.05	1.05
y'_0	1.15	1.15	1.15		

It can be seen from the figures that the RSEF model reproduces the central dip of rapidity distribution of heavier particles (proton and deuteron) in coincidence with the experimental findings, while for light particles (pions) there is a plateau instead of dip at central rapidity. Note that the appearance or disappearance of central dip is insensitive to the rapidity limit y'_{e0} but depends strongly on the magnitude of the

It can also be seen from Table I that $e_{\rm d} > e_{\rm p} > e_{\pi}$. It means that the system is less alongated for proton and deuteron than for pion. This describes nuclear stopping.

On the other hand, the width of the rapidity distributions are mianly controlled by the parameter $y'_{\rm e0}$. The value $y'_{\rm e0} = 1.35$, a little bigger than 1.15 used in the cylindrically-symmetrical flow model of Ref. [4] can account for the wide distriution of heavier particles (protons and deuterons) and at the same time fits the pion-distribution well.

In Fig.4 are shown the rapidity distributions of pions and protons for Au+Au collisions at 10.8 A GeV/c[5]. The solid and dashed lines correspond to the results of RSEF model (with parameters listed in Table I) and cylindrically-symmetrical flow model respectively. The latter are obtained also using RSEF with the same rapidity limit y'_0 as the $y'_{\rm e0}$ listed in Table I but with ellipicity $e = 1$. The histogram is the result from the RQMD model.

It can be seen from Fig.4 that in the RSEF model there is a shallow dip (plateau) in the central rapidity of the distribution of proton, instead of a central peak as predicted by the cylindrically-symmetrical flow model. However, the presently available experimental data are restricted to the large rapidity. The peak at central rapidity is the extrapolation of data using RQMD and is model dependent. It is intersting to see whether the prediction of a central dip (plateau) or a central peak will be observed in future experiments.

Comparing the parameter values for Si-Al (smaller colliding nuclei) and Au-Au (larger colliding nuclei) collisions listed in Table I, it can be seen that the rapidity limit $y'_{\rm e0}$ is smaller and the ellititicity e is bigger for the larger colliding nuclei than for the smaller ones. Both of these two show that the hadronic system formed from the larger colliding nuclei is less alongated, i.e. there is stronger nuclear stopping in the collision of larger nuclei.

IV. Summary and Conclusions

In high energy heavy-ion collisions, due to the transparency of the nucleus the participants will not lose the historical vestiges and the produced hadrons will carry some of their parent's memory of motion, leading to the unequivalence in longitudinal and transverse directions. So it is reasonable to assume that the flow of produced particle is privileged in the longitudinal direction. This picture has been used by lots of models[3,8]. Here we should mention two thermal and hydrodynamic models, one is the the boost-invariant longitudinal expansion model postulated by Bjorken [8] which can explain such an anisotropy already at the level of particle production. This model has been formulated for asymptotically high energies, where the rapidity distribution of produced particles establishes a plateau at midrapidity. The second model is the cylindrical symmetry flow model postulated first by Schnedermann, Sollfrank and Heinz[3]

circles. In this model the centers of fire-balls distribute uniformly in rapidity, and so it gives the picture that longitudinal and transverse expansion are totally independent. It can account for the wider rapidity distribution when comparing to the prediction of the pure thermal isotropically model but failed to reproduce the central dip in the proton and deuteron rapidity distributions.

In this paper, we propose an ellipsoidal expansion model with rotationally-symmetrical longitudinal flow (rotationally-symmetrical ellipsoidal flow model, RSEF) which realizes that the centers of fire-balls are distributed un-uniformly in a rotationally-symmetrical ellipsoidal shape around the longitudinal direction. The ellipticity parameter e can account for the extent of anisotropy of phase space in transverse and longitudinal expansion. The central dip in the proton and deuteron rapidity distributions and the central peak in the pion distribution are well reproduced simultaneously from this model.

It is found that the depth of the central dip of the heavier particle distributions depends strongly on the magnitude of the ellipicity e. In other words, the anisotropy in transverse and longitudinal directions of the ellipsiod of phase space, which is given by the ellipticity e, determines also the depth of the central dip for heavier particles.

Through comparing the feature of collision systems of different size, we found that the maximum flow velocities are larger for the lighter collision systems than the heavier ones, which suggests, together with smaller e, a larger stopping in the larger collision system.

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Figure captions

Fig.1 Schematic sketch of the cylindrically-symmetrical flow model.

Fig.2 Schematic sketch of the Rotationally-symmetrical ellipsoidal flow model

Fig.3 Rapidity distributions for central 14.6 A GeV/c Si+Al collisions. Open circles — data from Si+Al collisions [8,9]; Dashed lines — isotropical thermal model; Dotted lines — cylindrically-symmetrical flow model; solid line — rotationally-symmetrical ellipsoidal flow (RSEF) model. Temperature $T = 0.12GeV$. Fig.3 (a), (b) and (c) are for the pion, proton and deuteron distributions respectively.

Fig.4 Rapidity distributions for pions and protons in central Au+Au collisions with 10.8 A GeV/c. Full circles represent measured data, open circles reflected data. The solid line is our calculation using the RSEF model. The histogram shows the results from RQMD calculations and the dotted line is the results from the prediction of cylinderically-symmetrical flow model. The temperature $T = 0.14GeV$. Fig.4 (a) and Fig.4 (b) are for the pion and proton distributions respectively.

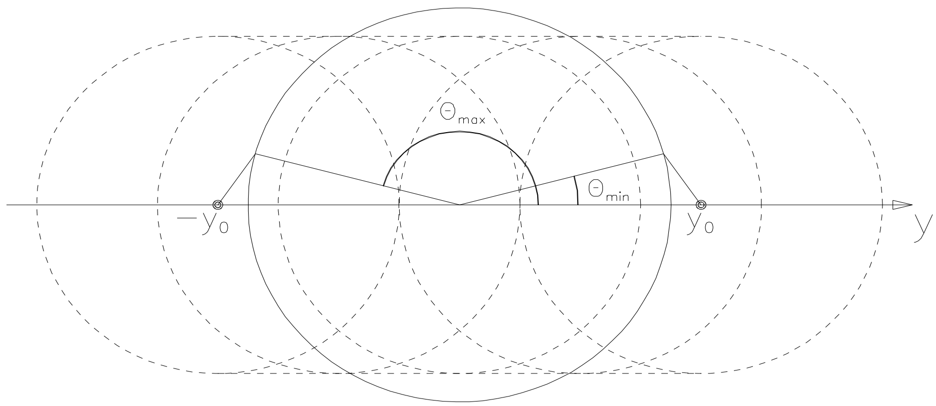

Fig. 1

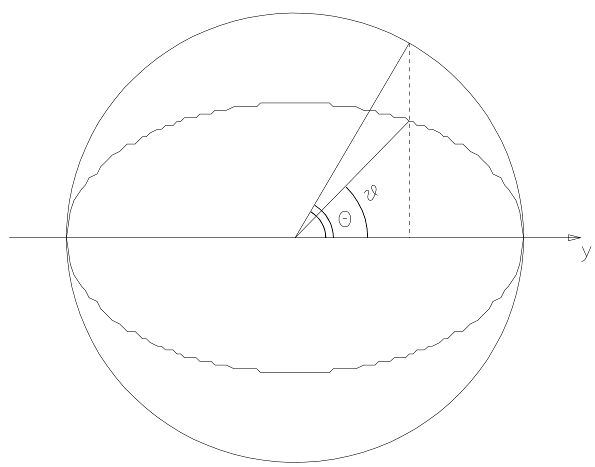

Fig. 2

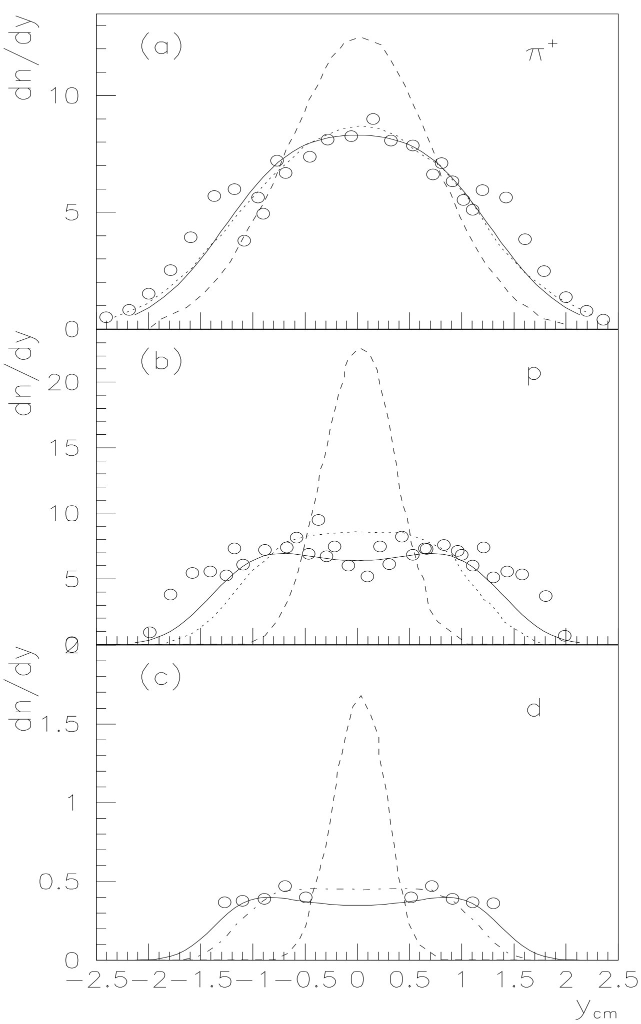

Fig. 3

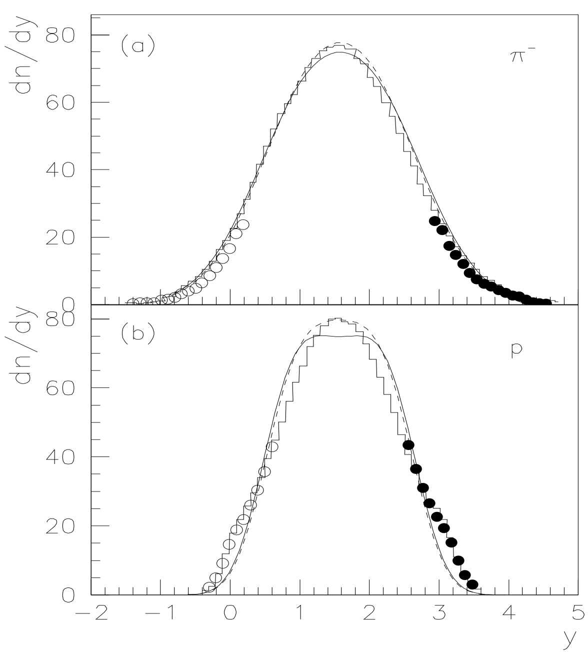

Fig. 4